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# VIBRATIONAL NONEQUILIBRIUM STAGNATION SHOCK LAYERS AT HYPERSONIC SPEED AND LOW REYNOLDS NUMBER†

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**Abstract**—The flow field near the stagnation streamline of a plane or axisymmetric blunt body is studied in the merged layer regime using the continuum approach. At the body surface the velocity slip and the jump of the translational and rotational temperature as well as that of the vibrational temperature are taken into account. The influence of the vibrational relaxation is discussed. Especially the wall temperature of an insulated body can be lowered considerably. Results are obtained not only for free flight conditions but also for wind tunnel tests. The latter agree with experimental results of other authors.

## NOMENCLATURE

$A, B$, constants in the series expansion of $u$;
$C^*$, constant in the viscosity relation;
$c_p^*$, isobaric specific heat;
$c_v^*$, isochoric specific heat;
$D$, constant in the viscosity relation, $D = D^*/T^*_{TR\infty}$;
$e$, internal energy, $e = e^*/V_\infty^{*2}$;
$j$, parameter of geometry ($j = 0$ for plane flow, $j = 1$ for axisymmetric flow);
$k$, coefficient of thermal conductivity, $k_{TR} = k^*_{TR}/k^*_{TR\infty}$;
$K_{\mathrm{I}}^*, K_{\mathrm{II}}^*$, constants relating to the relaxation length;
$Kn$, Knudsen number, $Kn = \Lambda^*/L^*$;
$L^*$, nose radius;
$M_\infty$, freestream Mach number at frozen vibration, $M_\infty = V_\infty^*/(\gamma R^* T^*_{TR\infty})^{1/2}$;
$p$, thermodynamic pressure, $p = p^*/\rho_\infty^* V_\infty^{*2}$;
$Pr_F$, Prandtl number at frozen vibration, $Pr_F = \mu^* c^*_{pTR}/k^*_{TR}$;
$Pr_E$, Prandtl number at vibrational equilibrium, $Pr_E = \mu^* c^*_{pE}/k^*_E$;
$\mathbf{q}$, heat flux vector, $\mathbf{q} = \mathbf{q}^*/\rho_\infty^* V_\infty^{*3}$;
$R^*$, specific gas constant;
$Re_\infty$, freestream Reynolds number, $Re_\infty = \rho_\infty^* V_\infty^* L^*/\mu_\infty^*$;
$St$, stagnation point Stanton number, $St = |\mathbf{q}^*(0,0)|/\{\rho_\infty^* V_\infty^*[c^*_{pTR}(T^*_{TR\infty} - T_w^*) + V_\infty^{*2}/2]\}$;
$T$, temperature, $T = T^*/T^*_{TR\infty}$;
$\mathbf{T}$, viscous stress tensor, $\mathbf{T} = \mathbf{T}^*/\rho_\infty^* V_\infty^{*2}$;
$u$, velocity component in $x$-direction;
$v$, velocity component in $y$-direction;
$\mathbf{v}$, vector of flow velocity, $\mathbf{v} = \mathbf{v}^*/V_\infty^*$;
$V_\infty^*$, absolute value of the freestream velocity, $V_\infty^* = |\mathbf{v}_\infty^*|$;
$w$, production rate of vibrational energy by vibrational excitation, $w = w^* L^*/V_\infty^{*3}$;
$x$, coordinate parallel to the body surface, $x = x^*/L^*$;
$y$, coordinate normal to the body surface, $y = y^*/L^*$.

Greek symbols

$\alpha$, thermal accommodation coefficient;
$\gamma$, ratio of specific heats at frozen vibration, $\gamma = c^*_{pTR}/c^*_{vTR}$;
$\vartheta$, angle between the body surface and the axis of symmetry;
$\theta$, characteristic temperature of vibration, $\theta = \theta^*/T^*_{TR\infty}$;
$\lambda$, relaxation length, $\lambda = \lambda^*/L^*$;
$\Lambda^*$, mean free path of the gas molecules;
$\mu$, coefficient of viscosity, $\mu = \mu^*/\mu_\infty^*$;
$\rho$, density, $\rho = \rho^*/\rho_\infty^*$;
$\sigma$, velocity accommodation coefficient.

Subscripts

$E$, vibrational equilibrium;
$TR$, translation and rotation;
$V$, vibration;
$w$, wall quantities;
$\infty$, freestream quantities.

Superscripts

$*$, dimensional quantities;
$'$, derivative with respect to $y$.

## 1. INTRODUCTION

THE FLIGHT of blunt-nosed re-entry vehicles in the upper atmosphere has been studied by many authors on various assumptions. In the present paper, the flow field in the vicinity of the stagnation streamline of a plane or axisymmetric body is investigated. Further, using the classification given by Hayes and Probstein [1], the flow is considered in the merged layer regime. Here, caused by the rarefaction, viscous effects are important in the entire region from the freestream to the body surface. But, in a good approximation, the gas may still be treated as a continuum.

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Such a problem has been studied by Kao [2] for an ideal gas without internal degrees of freedom. Chung, Holt and Liu [3] have considered the effect of non-equilibrium dissociation of a binary gas. Their investigations have been extended to a mixture of gases with relaxation of chemical reactions and ionization by Lee and Zierten [4]. An improvement has been obtained by Dellinger [5], who has taken into account the coupling of the ionization and the neutral gas properties.

In all these papers the slip velocity and the temperature jump have been neglected. Jain and Adimurthy [6] have shown, that these effects can play an important role also in the flow field in front of a highly cooled body. Further, these authors have found, that the thin-layer approximation used in [3–5] for example, is not justified for sufficiently rarefied flows. These calculations have demonstrated, that the gas may be treated as a continuum to relatively high freestream Knudsen numbers, if the complete Navier–Stokes equations are used together with the velocity slip and temperature jump boundary conditions. While in this work a pure ideal gas has been considered, the reacting mixture treated in [4] and [5] has been studied by Kumar and Jain [7] without the thin-layer approximation. Slip velocity and temperature jump have been included by Scott [8] and Hendricks [9].

In the present investigation, the effect of vibrational relaxation, which has been neglected in the papers cited above, is studied. A pure diatomic gas is considered, in which no other relaxation processes are important. The basic equations given in the next section are reduced to a system of nonlinear, coupled ordinary differential equations by using the concept of local similarity as discussed by Kao [2]. At the body surface the velocity slip and the jump of the translational and rotational temperature as well as that of the vibrational temperature are taken into account. The method of successive accelerated replacement is used for the numerical solution of the boundary-value problem.

Vibrational relaxation has also been taken into consideration by Obermeier [10] and by Tong [11]. In the latter paper the effect of assuming vibrational equilibrium has been examined. But the situations considered in both papers lie outside of the region of validity of the present study for two reasons. First, cases with discontinuous shocks have been treated, that is, no merged layer cases. Therefore, no slip and temperature jump conditions have been used at the body surface. Secondly, the relaxation of dissociation is taken into account, which may be neglected in the present cases. An extension of the present calculation to such cases, in which the dissociation should be considered, gives an estimation of the effect of vibrational nonequilibrium in these cases and allows a comparison with the results of Tong.

## 2. BASIC EQUATIONS

The stationary dimensionless balance equations for mass, momentum, total energy and vibrational energy of a diatomic gas are:

$$\operatorname{div}(\rho \mathbf{v}) = 0 \tag{1}$$

$$\rho \mathbf{v} \cdot \operatorname{grad} \mathbf{v} = -\operatorname{grad} p + \operatorname{div} \mathbf{T} \tag{2}$$

$$\rho \mathbf{v} \cdot \operatorname{grad}(e_{TR} + e_V + \mathbf{v}^2/2) = \operatorname{div}(-p\mathbf{v} + \mathbf{v} \cdot \mathbf{T}) - \operatorname{div}(\mathbf{q}_{TR} + \mathbf{q}_V) \tag{3}$$

$$\rho \mathbf{v} \cdot \operatorname{grad} e_V = -\operatorname{div} \mathbf{q}_V + \rho w. \tag{4}$$

The gas is assumed to be thermally perfect, that is

$$p = \rho T_{TR}/\gamma M_\infty^2 \tag{5}$$

and calorically perfect at frozen vibration, that is

$$e_{TR} = T_{TR}/\gamma(\gamma - 1)M_\infty^2 \tag{6}$$

with constant ratio $\gamma$ of the specific heats at frozen vibration.

It is assumed, that the distribution of the oscillators over the vibrational energy states is a Boltzmann distribution. Then, the dependence of the vibrational energy on the vibrational temperature is given by

$$e_V = (\theta/\gamma M_\infty^2)/[\exp(\theta/T_V) - 1]. \tag{7}$$

Following the theory of Landau and Teller (see Vincenti and Kruger [12]), in which the molecules are treated as harmonic oscillators, the production rate of vibrational energy is

$$w = (e_{VE} - e_V)/\lambda. \tag{8}$$

For the treatment of stationary flows, the relaxation length $\lambda$ is more convenient than a relaxation time. Following Vincenti and Kruger, at sufficiently low temperatures, the following approximation can be used:

$$\lambda = (K_{\mathrm{I}}^*/Re_\infty \mu_\infty^* p)\exp[(K_{\mathrm{II}}^*/T_{TR\infty}^* T_{TR})^{1/3}]. \tag{9}$$

The values f the parameters $K_{\mathrm{I}}^*$ and $K_{\mathrm{II}}^*$ depend on the properties of the special gas.

The coefficient of viscosity is assumed to be given by Sutherland's formula

$$\mu^* = C^*(T_{TR}^*)^{1/2}/(1 + D^*/T_{TR}^*)$$

with the dimensionless form

$$\mu = (1 + D)(T_{TR})^{1/2}/(1 + D/T_{TR}). \tag{10}$$

The flux of translational and rotational energy is taken to be given by Fourier's law, and an analogous behaviour of the flux of vibrational energy is assumed:

$$\mathbf{q}_{TR} = -k_{TR} \operatorname{grad} T_{TR}/(\gamma - 1)M_\infty^2 Re_\infty Pr_{F\infty} \tag{11}$$

$$\mathbf{q}_V = -k_V \operatorname{grad} T_V/(\gamma - 1)M_\infty^2 Re_\infty Pr_{F\infty}. \tag{12}$$

The Prandtl numbers $Pr_F$ at frozen vibration and $Pr_E$ at vibrational equilibrium are assumed to be equal to the same constant:

$$Pr_F \equiv Pr_\infty, \quad Pr_E \equiv Pr_\infty. \tag{13}$$

The values of $\mu^*$ and $c_{pTR}^*$ being known, the coefficient $k_{TR}$ of conductivity of translational and rotational energy can be calculated from the first of these equations. Since at vibrational equilibrium

$$k_E = k_{TR} + k_{VE}$$

and

$$c^*_{pE} = c^*_{pTR} + \frac{de^*_{VE}}{dT^*_{TR}}$$

equation (13) leads to

$$k_{VE}/k_{TR} = (\gamma-1)M_\infty^2 \frac{de_{VE}}{dT_{TR}}.$$

Here it is assumed, that an analogous relation holds at vibrational relaxation:

$$k_V/k_{TR} = (\gamma-1)M_\infty^2 \frac{de_V}{dT_V}. \tag{14}$$

In the following, the flow shall be studied in the vicinity of the stagnation streamline of blunt bodies. Plane or axisymmetric geometry is assumed. The flow variables are expanded into series about the axis of symmetry. Following the concept of local similarity, these series are truncated. Thus, in the boundary-layer coordinate system used here:

$$\left.\begin{aligned} &u(x,y) = xu_1(y) + x^3\{Au_1(y)+B\}\\ &v(x,y) = (1-x^2/2+x^4/24)v_0(y)\\ &p(x,y) = p_0(y)+x^2p_2(y)\\ &\rho(x,y) = \rho_0(y)\\ &T_{TR}(x,y) = \{1+x^2p_2(y)/p_0(y)\}T_{TR0}(y)\\ &\mu(x,y) = \{1+x^2p_2(y)[1+3D/T_{TR0}(y)]/\\ &\qquad 2p_0(y)[1+D/T_{TR0}(y)]\}\mu_0(y)\\ &T_V(x,y) = T_{V0}(y). \end{aligned}\right\} \tag{15}$$

These equations are slightly different from those used by other authors (see Hayes and Probstein [1], for example). First, the expansions of $u$ and $v$ contain all terms, which, without truncation, appear in the differential equations used below. Also the boundary conditions are met to this order. Secondly, the expansions of $T_{TR}$ and $\mu$ are taken to be consistent with the material equations (5) and (10). As stated by Hayes and Probstein, full self-consistency cannot be obtained. Here, the boundary condition for $T_{TR}$ can be met only in the lowest order.

With use of (8) and (11)–(14) and with introduction of the boundary-layer coordinates and of (15) the balance equations reduce to a set of ordinary differential equations. These are not given here, since the calculation is straightforward and the results are rather lengthy.

### 3. BOUNDARY CONDITIONS

At the free stream, all quantities are taken to be constant

$$\left.\begin{aligned} &u(x,y) = \cos\vartheta \qquad T_{TR}(x,y) \equiv 1\\ &v(x,y) = -\sin\vartheta \qquad T_V(x,y) \equiv T_{V\infty}\\ &\rho(x,y) \equiv 1. \end{aligned}\right\} \tag{16}$$

These conditions are met at an effective freestream distance $y_A$, which is not known *a priori*.

There are two possible cases concerning the value of $T_{V\infty}$. First, a freestream state can be constant, if vibrational equilibrium is given, that is $T_{V\infty} = 1$. Secondly, a constant freestream state is possible at vibrational nonequilibrium, that is at $T_{V\infty} \neq 1$, if the vibration is frozen in the free stream. This occurs, if the freestream relaxation length $\lambda_\infty$ is much greater than the effective freestream distance. Such situations are given during the simulation of hypersonic flight in a wind tunnel.

On the body surface, first order velocity slip and temperature jump conditions are used (see Emmons [13], for example):

$$\left.\begin{aligned} &u(x,0) = \left(\frac{2-\sigma}{\sigma}Kn\frac{\partial u}{\partial y} + \frac{3}{2(2\pi)^{1/2}}\frac{Kn}{(\gamma)^{1/2}M_\infty}\frac{1}{(T_{TR})^{1/2}}\frac{\partial T_{TR}}{\partial x}\right)_{y=0}\\ &v(x,0) = 0\\ &T_{TR}(x,0) = T_w + \left(\frac{2-\alpha_{TR}}{\alpha_{TR}}\frac{2\gamma}{\gamma+1}\frac{Kn}{Pr_\infty}\frac{\partial T_{TR}}{\partial y}\right)_{y=0}. \end{aligned}\right\} \tag{17}$$

With use of (6) the last of these equations may be written as a jump condition for the internal energy $e_{TR}$. At vibrational equilibrium an analogous jump condition is valid for the total internal energy $(e_{TR}+e_{VE})$. From these two conditions a jump condition for $e_{VE}$ can be derived. It is assumed, that the same condition is valid for $e_V$ at vibrational nonequilibrium. Thus:

$$e_V(x,0) = e_{VE}(T_w) + \left(F\frac{2-\alpha_V}{\alpha_V}\frac{Kn}{Pr_\infty}\frac{\partial e_V}{\partial y}\right)_{y=0} \tag{18}$$

with

$$\left.\begin{aligned} &F = 2[(\gamma^2+1)/(\gamma+1)+c^*_V/c^*_{vTR}]/(\gamma+1+2c^*_V/c^*_{vTR})\\ &c^*_V = de^*_V/dT^*_V. \end{aligned}\right\} \tag{19}$$

Since it can be shown, that $1 \leqslant F \leqslant 1.03$, in the following, the approximation

$$F = 1 \tag{20}$$

is used. Then, equation (18) coincides with the jump condition given by Mitra and Fiebig [14] without derivation.

If the body surface is cooled, its temperature $T_w$ is prescribed. If the wall is adiabatic, the total normal heat flux must be zero, that is:

$$\left(k_{TR}\frac{\partial T_{TR}}{\partial y}+k_V\frac{\partial T_V}{\partial y}\right)_{y=0}=0. \qquad (21)$$

Introduction of the series expansions (15) into these boundary conditions leads to conditions for the expansion coefficients.

## 4. NUMERICAL TREATMENT

The set of differential equations to be solved is a coupled set of nonlinear first- and second-order differential equations. Here, the method of successive accelerated replacement has been employed for the solution of the second-order equations. This method has been used by Dellinger [5], Jain and Adimurthy [6] and Hendricks [9] previously. It consists in the improvement of certain initial guesses by iteration. The first-order equations have been integrated by numerical quadrature for every new approximation.

All calculations have been done for nitrogen with the following values of the constants:

$$Pr_\infty = 0.75, \quad \gamma = 1.4,$$
$$C^* = 1.398\cdot 10^{-5}\,\mathrm{g/cm\,s\,K^{1/2}}, \quad D^* = 103\,\mathrm{K},$$
$$\theta^* = 3390\,\mathrm{K}, \quad K_I^* = 7.12\cdot 10^{-3}\,\mathrm{g/cm\,s},$$
$$K_{II}^* = 1.91\cdot 10^6\,\mathrm{K}.$$

The values of $C^*$ and $D^*$ result from adjusting measured values of $\mu^*$ to Sutherland's viscosity law. The values of $\theta^*$, $K_I^*$ and $K_{II}^*$ are given by Vincenti and Kruger [12].

The accommodation coefficients have been taken to be

$$\sigma = 1, \quad \alpha_{TR} = 1, \quad \alpha_V = 1 \quad \text{and} \quad \alpha_V = 0.001,$$

since the order of magnitude of $\sigma$ and $\alpha_{TR}$ usually is 1, while $\alpha_V$ may be very small (see Mitra and Fiebig [14]).

## 5. REGION OF VALIDITY

In the preceeding sections several basic assumptions have been used. First, the gas has been treated as a continuum. This is a good approximation only at moderate Knudsen numbers, that is

$$Kn_\infty \leqslant 0.1. \qquad (22)$$

Further, the dissociation of the gas molecules has been neglected. In order to check the validity of this assumption, an approximate consideration given by Harney [15] has been applied to the present case. Following this estimation, the freestream density has to be less than a critical value:

$$\rho_\infty^* \leqslant \rho_{\infty cr}^*(V_\infty^*, L^*). \qquad (23)$$

Figure 1 shows $\rho_{\infty cr}^*$ for some values of the nose radius.

At last, the concept of local similarity was used. This leads to a good approximation only for hypersonic flow, that is for

$$M_\infty \geqslant 10. \qquad (24)$$

The conditions (22)–(24) form the boundaries of the region of validity of the present calculations. Figure 2

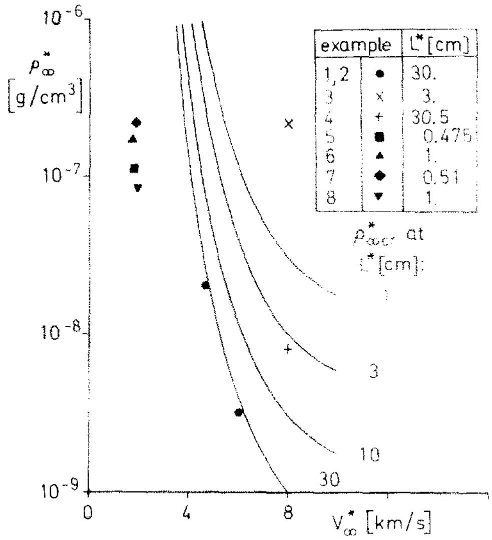


FIG. 1. Criterion for neglecting the dissociation.

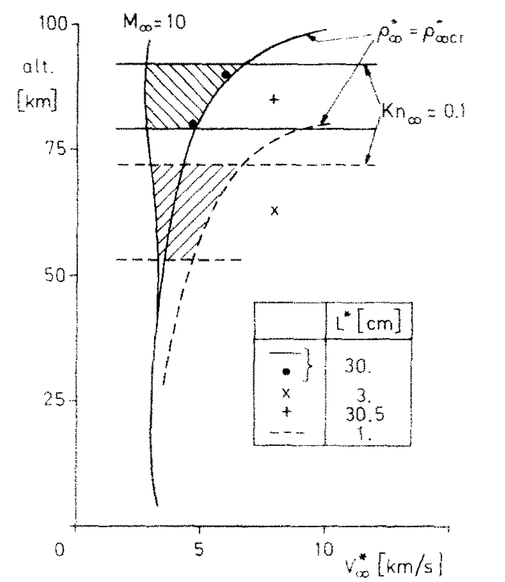


FIG. 2. Validity of the present calculation for atmospheric flight conditions.

shows this region for a body moving in the upper atmosphere. The lower boundaries of the shaded areas are caused by the fact, that the numerical procedure does not work very well, if steep gradients, that is nearly unsteady shocks, form in front of the body. The consideration of such cases lies outside of the scope of the present paper.

## 6. DISCUSSION OF THE RESULTS

Two sets of numerical examples have been considered. The freestream conditions chosen for examples 1–4 listed in Table 1 occur at the flight in the upper atmosphere. The Figs. 1 and 2 show, that the present approximations are valid for the examples 1 and 2, while for the other two examples the dissociation should not be neglected. In these cases nevertheless certain conclusions may be drawn from

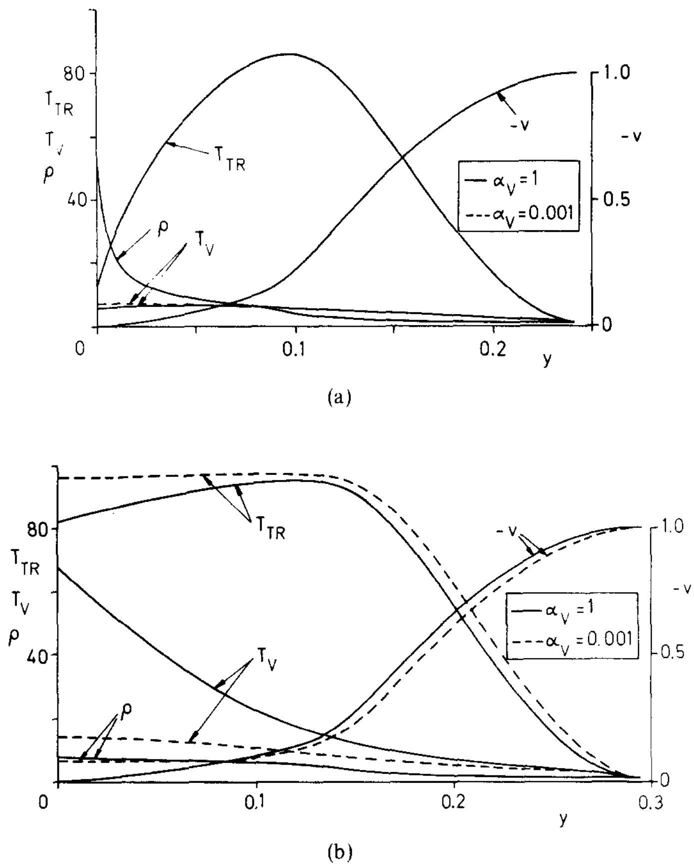


FIG. 3. Distribution of the flow variables along the stagnation streamline for example 1: (a) for cold wall; (b) for insulated wall.

Table 1. Examples for free flight conditions

| Number of example | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $j$ | 1 | 1 | 1 | 1 |
| $L^*$ [cm] | 30 | 30 | 30.5 | 3 |
| $V^*_\infty$ [km/s] | 6 | 4.7 | 7.94 | 7.94 |
| $\rho^*_\infty$ [$10^{-8}$ g/cm$^3$] | 0.32 | 2 | 0.8 | 21.5 |
| $T^*_{TR\infty}$ [K] | 180 | 180 | 180 | 247 |
| $T^*_w$ [K] for cooled wall | 1000 | 1000 | 1500 | 1500 |
| $M_\infty$ | 21.9 | 17.2 | 29.0 | 24.8 |
| $Re_\infty$ | 483 | 2364 | 1624 | 3300 |
| $Kn_\infty$ | 0.068 | 0.011 | 0.027 | 0.011 |

Table 2. Examples for test conditions in a wind tunnel

| Number of example | 5 | 6 | 7 | 8 |
|---|---|---|---|---|
| $j$ | 0 | 1 | 0 | 1 |
| $L^*$ [cm] | 0.475 | 1 | 0.51 | 1 |
| $V^*_\infty$ [km/s] | 1.737 | 1.764 | 1.845 | 1.966 |
| $\rho^*_\infty$ [$10^{-8}$ g/cm$^3$] | 11.0 | 17.37 | 21.92 | 8.26 |
| $T^*_{TR\infty}$ [K] | 18.5 | 13.7 | 14.4 | 22.4 |
| $T^*_{V\infty}$ [K] | 1530 | 1470 | 1650 | 1840 |
| $T^*_w$ [K] | 290 | 302 | 300 | 300 |
| $M_\infty$ | 19.8 | 23.4 | 23.8 | 20.4 |
| $Re_\infty$ | 991 | 5065 | 3158 | 1378 |
| $Kn_\infty$ | 0.030 | 0.007 | 0.011 | 0.022 |

the results. The freestream conditions chosen for examples 5–8 presented in Table 2 are typical for the simulation of the free flight in a wind tunnel. In these cases not only the conditions (22) and (24) are fulfilled, but also condition (23), which is shown in Fig. 1.

### 6.1. *Results for free flight conditions*

In Fig. 3a the distribution of some flow variables along the stagnation streamline is presented for example 1 at a highly cooled wall. In this case the vibrational temperature remains nearly at its freestream level. A small influence of the different values taken for $\alpha_V$ on the vibrational temperature can be seen, while the influence on the remaining quantities is even smaller. The effect of the vibrational relaxation on the stagnation point heat transfer is demonstrated in Table 3. Comparison with the result for frozen vibration shows a difference of at most 0.5% for the first example.

With increasing wall temperature at one hand the region of high translational and rotational temperature becomes larger. Thus the vibrational temperature has the opportunity to adjust to the translational and rotational temperature over a larger distance. Consequently the vibrational temperature can reach a higher level. At the other hand with increasing wall temperature the vibrational temperature near the wall increases. These facts are demonstrated in Fig. 3b for

Table 3. Stagnation point Stanton number

| Number of example | Case | Contribution of translation and rotation | Contribution of vibration | Total |
|---|---|---|---|---|
| 1 | *A* | 0.389 | 0.0011 | 0.390 |
| | *B* | 0.391 | 0.00001 | 0.391 |
| | *C* | 0.392 | 0 | 0.392 |
| 2 | *A* | 0.167 | 0.0034 | 0.170 |
| | *B* | 0.167 | 0.0001 | 0.167 |
| | *C* | 0.167 | 0 | 0.167 |
| 3 | *A* | 0.236 | 0.014 | 0.250 |
| | *B* | 0.242 | 0.0002 | 0.242 |
| | *D* | 0.207 | 0.050 | 0.257 |
| 4 | *A* | 0.131 | 0.022 | 0.153 |
| | *B* | 0.133 | 0.0004 | 0.133 |
| | *D* | 0.127 | 0.028 | 0.155 |
| 6 | *A* | 0.270 | 0.023 | 0.293 |
| | *B* | 0.275 | 0.0002 | 0.275 |
| | *E* | | | 0.263 |
| 7 | *A* | 0.240 | 0.025 | 0.265 |
| | *B* | 0.242 | 0.0002 | 0.242 |
| | *E* | | | 0.215 |
| 8 | *A* | 0.390 | 0.044 | 0.434 |
| | *B* | 0.390 | 0.0003 | 0.390 |
| | *E* | | | 0.382 |

Notation: *A*, vibrational relaxation, $\alpha_V = 1$; *B*, vibrational relaxation, $\alpha_V = 0.001$; *C*, frozen vibration; *D*, vibrational equilibrium; *E*, results of measurements.

an insulated wall. At a small value of the accommodation coefficient $\alpha_V$ only the first effect is of importance. At $\alpha_V = 1$ also the second effect becomes relevant. It leads to a relatively high vibrational temperature and also to a markable change in the other flow variables. Especially near the wall the translational and rotational temperature is lowered, since a certain amount of energy is transferred to the vibrational degree of freedom. The resulting temperature of the insulated wall is given in Table 4.

In Fig. 4 the temperatures for example 2 are presented at a cooled and at an insulated wall. For this example, due to the lower Knudsen number, the region of the shock structure is smaller than for example 1. Thus, the translational and rotational temperature has reached its maximum at a distance from the stagnation point which, compared to the extension of the whole shock layer, is much greater than for example 1. Therefore, the vibrational temperature has more time to rise, and consequently it reaches somewhat higher values. At a highly cooled wall, nevertheless, there is a

Table 4. Temperature of the insulated wall

| Number of example | Case | $T_w$ |
|---|---|---|
| 1 | *A* | 78 |
| | *B* | 96 |
| | *C* | 98 |
| 2 | *A* | 49 |
| | *B* | 57 |
| | *C* | 60 |

marked influence of the different values taken for $\alpha_V$ only on the vibrational temperature. The difference of the stagnation point heat transfer from the result for frozen vibration reaches 1.8%. Since the error introduced by the approximations used in the present calculations is probably higher than this value, the vibration may be treated as frozen. This is no longer justified at higher wall temperatures. For, especially at an insulated wall, as for example 1, the wall temperature can be lowered considerably by vibrational relaxation. The difference to the value for frozen vibration depends strongly on the accommodation coefficient $\alpha_V$.

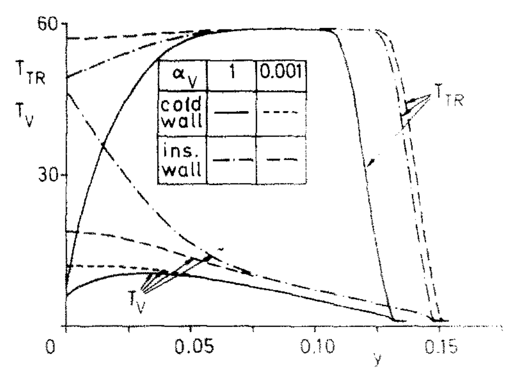


FIG. 4. Temperature profiles along the stagnation streamline for example 2.

When the dissociation is not negligible, that is, when the condition (23) is not fulfilled, calculations are usually done on the assumption of vibrational equilibrium. The validity of this assumption has been tested by Tong [11] for a binary gas model with vibration-dissociation coupling. Since in most of the examples considered by Tong the Knudsen number is very small, he has used the approximation of a discontinuous shock. Further, since he has neglected the flux of vibrational energy, his balance equation for the vibrational energy is of first order, and no boundary condition at the wall can be prescribed. Therefore, during the integration from the shock to the body, he has assumed vibrational equilibrium from a point, in which the vibrational energy has approached a certain percentage of the equilibrium value. Though only an estimation of the effect of vibrational relaxation was intended by Tong, the second approximation could have some influence on the heat transfer. Since in the present calculation the dissociation is neglected, it should not be extended to the region considered by Tong. Nevertheless, the examples 3 and 4 were chosen in order to give at least an estimation of the effect of vibrational relaxation. Comparison of the resulting values of the heat transfer shows, that vibrational relaxation may change this value by 6% or 14%, respectively. Example 3 corresponds to one of the examples given by Tong, who also found an influence of at most 6%. Example 4 indicates, that, following this rough estimation, even a greater error is possible, when the assumption of vibrational equilibrium is used.

Figure 5 shows the influence of this assumption on the temperatures.

### 6.2. *Results for test conditions in a wind tunnel*

In Fig. 6 the temperatures calculated for example 5 are presented together with some experimental values measured by Bütefisch [16]. Here, the influence of the different values taken for $\alpha_V$ on the vibrational temperature is quite important, while the influence on the translational and rotational temperature is not visible.

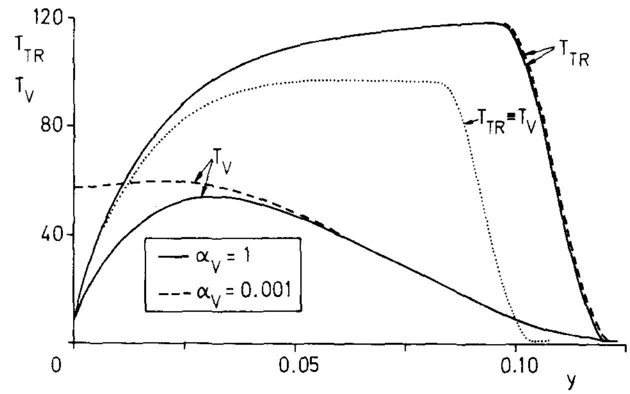


FIG. 5. Change of the temperature profiles for example 4 by the assumption of vibrational equilibrium.

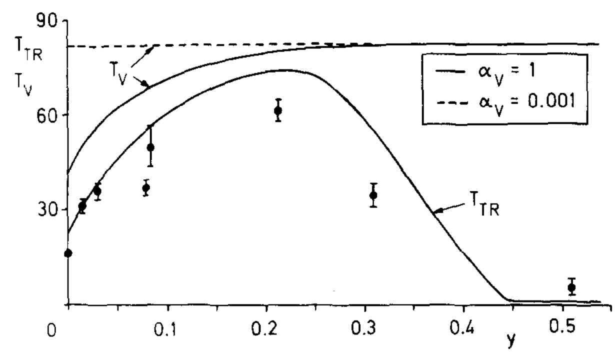


FIG. 6. Temperature profiles along the stagnation streamline for example 5. Ī Values of $T_{TR}$ measured by Bütefisch [1].

Regarding the spread of the experimental values of the translational and rotational temperature, in the region near the wall, the agreement between the calculated and the measured results is quite satisfying. Near the freestream region, the agreement is not so good. This may be caused by the fact, that the calculation with the continuum approximation cannot give the exact structure of the flow field over a distance, which is smaller than the mean free path of the molecules, the latter being relatively long in this region.

The examples 6–8 were chosen in order to compare the stagnation point heat transfer with values measured by Vennemann [17] (examples 6 and 8) and Koppenwallner [18] (example 7). Table 3 shows, that the different values taken for $\alpha_V$ lead to a difference in the Stanton number of 6.5%, 9.5%, and 11.3%, respectively. The value of $\alpha_V$ prevailing at the experiments is not known, but this quantity may be very small. Agreement between the heat transfer calculated for $\alpha_V = 0.001$ and the measured value is given, when an error of at most 6% on both the calculations and the experiments is admitted.

## 7. CONCLUSIONS

The influence of vibrational relaxation on the viscous flow in the vicinity of the stagnation streamline of a blunt body has been studied.

On free flight conditions, only a small effect of the variation of vibrational energy on the heat transfer to a highly cooled body has been found. With increasing wall temperature the dependence of all flow quantities on the vibrational relaxation has been shown to increase. Especially the wall temperature of an insulated body can be lowered considerably. The intensity of these effects depends on the completeness of the accommodation of the vibrational energy at the wall.

An extension of the calculation to cases, in which the dissociation should be taken into account, can give but an estimation of the influence of the vibrational relaxation. This estimation has led to the result, that a relatively great error may be introduced by the assumption of vibrational equilibrium usually employed in these cases.

On test conditions in a wind tunnel, that is, on nonequilibrium freestream conditions, the vibrational temperature and the heat transfer have turned out to depend strongly on the accommodation of the vibrational energy. The results of the calculations agree with experimental results of other authors.

## REFERENCES

1. W. D. Hayes and R. F. Probstein, *Hypersonic Flow Theory*, Chapter 10. Academic Press, New York (1959).
2. H. C. Kao, Hypersonic viscous flow near the stagnation streamline of a blunt body, Part 1: A test of local similarity, Part 2: Third-order boundary-layer theory and comparison with other methods, *AIAA Jl* **2**, 1892–1906 (1964).
3. P. M. Chung, J. F. Holt and S. W. Liu, Merged stagnation shock layer of a nonequilibrium dissociating gas, *AIAA Jl* **6**, 2372–2379 (1968).
4. R. H. C. Lee and T. A. Zierten, Merged layer ionization in the stagnation region of a blunt body, *Proceedings of the 1967 Heat Transfer and Fluid Mechanics Institute*, pp. 452–468. Stanford University Press, Stanford (1967).
5. T. C. Dellinger, Computation of nonequilibrium merged stagnation shock layers by successive accelerated replacement, *AIAA Jl* **9**, 262–269 (1971).
6. A. C. Jain and V. Adimurthy, Hypersonic merged stagnation shock layers, *AIAA Jl* **12**, 342–354 (1974).
7. A. Kumar and A. C. Jain, Nonequilibrium merged stagnation shock layers at hypersonic speeds, *Int. J. Heat Mass Transfer* **18**, 1113–1118 (1975).
8. C. D. Scott, Reacting shock layers with slip and catalytic boundary conditions, *AIAA Jl* **13**, 1271–1278 (1975).
9. W. L. Hendricks, A similarity solution of the Navier–Stokes equations with wall catalysis and slip for hypersonic, low Reynolds number flow over spheres, AIAA-Paper No. 75-675, AIAA 10th Thermophysics Conference, Denver, Colorado (1975).
10. E. Obermeier, Relaxations- und Transporterscheinungen bei der Hyperschallströmung um stumpfe Körper, Dissertation, Bochum, Germany (1972).
11. H. Tong, Effects of dissociation energy and vibrational relaxation on heat transfer, *AIAA Jl* **4**, 14–18 (1966).
12. W. G. Vincenti and C. H. Kruger, *Introduction to Physical Gas Dynamics*, Chapter 7. Wiley, New York (1965).
13. H. W. Emmons (editor), *Fundamentals of Gas Dynamics, High Speed Aerodynamics and Jet Propulsion*, Vol. 3, pp. 718–719. Princeton University Press, New Jersey (1958).
14. N. K. Mitra and M. Fiebig, Low Reynolds number vibrational nonequilibrium flow in Laval nozzle, in *Rarefied Gas Dynamics, Proceedings of the 9th International Symposium* 1974, pp. B.21-1–B.21-12, DFVLR-Press, Porz-Wahn (1974).
15. D. J. Harney, Chemical kinetic regimes of hypersonic flight simulation, Arnold Engng Dev. Center, TDR-63-3 (1963).
16. K. A. Bütefisch, Experimentelle Bestimmung von Rotations- und Schwingungstemperatur und Dichte mit Hilfe der Elektronenstrahltechnik im Strömungsfeld eines hypersonisch angeströmten Kreiszylinders bei Stautemperaturen von 1100–1800 K, Deutsche Luft und Raumfahrt FB 69-63, Göttingen, Germany (1969).
17. D. Vennemann, Messungen des konvektiven Wärmeübergangs an einem Zylinder mit Halbkugelkopf in verdünnter hypersonischer Strömung, AVA-IB 063-72 H 05, Aerodynamicshe Versuchsanstalt, Göttingen, Germany (1972).
18. G. Koppenwallner, Heat transfer to circular cylinders in hypersonic rarefied gas flow, AVA-IB 69 A 33, Aerodynamische Versuchsanstalt, Göttingen (1969). Paper presented et EUROMECH 13, The Aerodynamics of Rarefied Gas Flow, Teddington, Great Britain (1969).


## ECOULEMENT D'UN GAZ HORS D'EQUILIBRE VIBRATOIRE PRES D'UN POINT D'ARRET A VITESSE HYPERSONIQUE ET A UN NOMBRE DE REYNOLDS FAIBLE


**Résumé**—L'écoulement près de la ligne de courant passant par le point d'arrêt d'un corps émoussé plan ou de révolution est étudié dans le régime de mélange du choc et de la couche limite. L'approximation continue est employée. A la surface du corps, il est tenu compte non seulement du glissement et de la discontinuité de la température de translation et de rotation mais encore de la discontinuité de la température de vibration. L'influence de la relaxation vibratoire est discutée. En particulier, la température de la paroi d'un corps athermane peut être diminuée considérablement. Des résultats sont donnés non seulement pour les conditions de mouvement dans l'atmosphère mais encore pour les conditions d'essai en soufflerie. Dans le dernier cas, les résultats sont en accord avec les expériences d'autres auteurs.


## STAUPUNKTSSTRÖMUNG MIT NICHTGLEICHGEWICHT DER SCHWINGUNG BEI HYPERSCHALLGESCHWINDIGKEIT UND KLEINER REYNOLDSZAHL


**Zusammenfassung**—Das Strömungsfeld in der Umgebung der Staupunktsstromlinie eines ebenen oder rotationssymmetrischen stumpfen Körpers wird für den Fall der Vermischung von Verdichtungsstoß und Grenzschicht unter Benutzung des Kontinuumsmodells für das Gas untersucht. An der Körperoberfläche werden die Effekte des Gleitens und des Sprunges der Translations- und Rotationstemperatur sowie desjenigen der Schwingungstemperatur berücksichtigt. Der Einfluß der Schwingungsrelaxation wird diskutiert. Insbesondere kann dadurch die Wandtemperatur eines isolierten Körpers beträchtlich erniedrigt werden. Nicht nur für Bedingungen beim freien Flug sondern auch für solche bei Windkanaluntersuchungen werden Ergebnisse angegeben. Diese stimmem für den letzteren Fall mit experimentellen Ergebnissen anderer Autoren überein.


## КОЛЕБАТЕЛЬНЫЕ НЕРАВНОВЕСНЫЕ УДАРНЫЕ СЛОИ ТОРМОЖЕНИЯ ПРИ СВЕРХЗВУКОВЫХ СКОРОСТЯХ И МАЛЫХ ЧИСЛАХ РЕЙНОЛЬДСА


**Аннотация** — Методом сплошных сред исследуется распределение параметров потока в окрестности точки при обтекании плоского или осесимметричного затупленного тела в «режиме поглощающего слоя». Учитывается скорость скольжения на поверхности тела и изменение поступательной, вращательной, а также колебательной температуры на стенке. Рассматрииается влияние колебательной релаксации. В частности, температура стенки изолированного тела может быть значительно уменьшена. Результаты получены не только для условий свободного полета, но и для опытов в аэродинамической трубе. Последние хорошо согласуются с экспериментальными данным других авторов.